

# Rescaling pressure and surface energy for the contact mechanics challenge

Some experimentalists expressed interest in entering the contact mechanics challenge. An impediment to this plan might be that they cannot simply set the dimensions of the system as defined in the challenge or that they use a material with a contact modulus differing from that stated in the problem definition. To help those scientists in setting up their experiments, some formulas are provided on how to redefine the problem such that all relevant dimensionless numbers remain unchanged.

The contact mechanics challenge is defined for a linear system size of 0.1 mm and a root-mean-square surface-height gradient of  $\bar{g} = 1$ . The contact modulus is chosen as  $E^* = 25$  MPa, the surface energy as  $\gamma = 50$  mJ/m<sup>2</sup>, and the externally applied pressure to  $p_0 = 250$  kPa. We now assume that potentially participating experimentalists attempt to build surfaces that are scaled versions of the proposed ones and that their materials have a different contact modulus. If the defined parameters are scaled as follows

$$\lambda_s \rightarrow s_x \lambda_s \quad (1)$$

$$h(\mathbf{r}) \rightarrow s_z h(\mathbf{r}) \quad (2)$$

$$E^* \rightarrow s_E E^*, \quad (3)$$

i.e., the new values of  $\lambda_s$  is equal to the old one times  $s_x$ , etc., then how do load, surface energy and interaction range need to be rescaled?

The new root-mean-square gradient is

$$\bar{g}_{\text{new}} = \frac{s_z}{s_x}. \quad (4)$$

One should make sure that  $\bar{g}_{\text{new}}$  does not exceed (much) the original value of  $\bar{g} = 1$ , because otherwise the small-slope approximation becomes poor. Since pressure is best expressed in units of  $E^* \bar{g}$ , the new default value for the external pressure is obtained with

$$p_0 \rightarrow s_E \bar{g}_{\text{new}} p_0. \quad (5)$$

We expect that most experimentalist would simply scale up the surface isotropically, in which case they only need to multiply the mean contact pressure by  $s_E$ . It shall be left to the experimentalist to convert the pressures into a load.

At this point, it might be worth saying a word of caution. In the simulations, we use periodic boundary conditions, while experiments will not do that. Instead, one

replica will probably have to do the job. This means that the coarse-grained pressure distribution within the contact can differ between experiment and simulation in a non-negligible fashion, unless precaution is taken. One possibility to make them similar is to use a thickness  $t$  of the loaded elastic manifold that is less than the linear system size  $\mathcal{L}$  but still more than the roll-off wavelength  $\lambda_r$ . A reasonable compromise might be to use a thickness of  $t \approx \sqrt{\mathcal{L} \lambda_r}$ .

The rescaling of the surface energy is the last term needing discussion. First, the range of adhesion should be “short”, i.e.,

$$\rho_{\text{new}} \lesssim s_z \cdot 2 \text{ nm}. \quad (6)$$

Second, given the definition of the dimensionless surface energy,  $\tilde{\gamma}$ , see equation (10) in reference 1, for randomly rough surfaces

$$\tilde{\gamma} = \frac{\gamma}{E^* R_c} \frac{1}{\bar{g}^3}, \quad (7)$$

$R_c$  being the characteristic local surface curvature, one gets

$$\frac{\gamma_{\text{new}}}{\gamma} = \frac{E_{\text{new}}^*}{E^*} \frac{R_{c,\text{new}}}{R_c} \frac{\bar{g}_{\text{new}}^3}{\bar{g}^3} \quad (8)$$

$$= s_E \frac{s_x^2}{s_z} \frac{s_z^3}{s_x^3}, \quad (9)$$

or,

$$\gamma \rightarrow s_E \frac{s_z^2}{s_x} \gamma. \quad (10)$$

<sup>1</sup> M. H. Müser, A dimensionless measure for adhesion and effects of the range of adhesion in contacts of nominally flat surfaces, Tribol. Int. (available online), DOI: